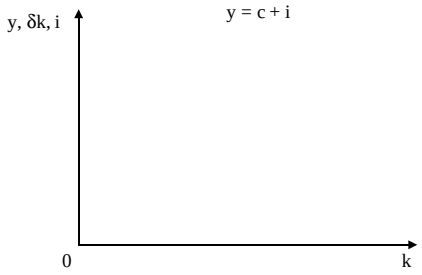
[slides for Part (b) of this topic here]

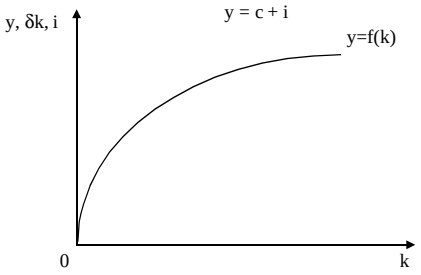
The golden rule

- The golden rule level of capital accumulation k^*_{gold}
- That level of **steady-state** capital which implies maximum **steady-state** consumption c^*_{gold}
- In this version, given by MPK = δ
- All golden rule equilibria are steady-states, but...
- not all steady-states are golden rules...

Solow Growth Model: The golden rule



Solow Growth Model: The golden rule



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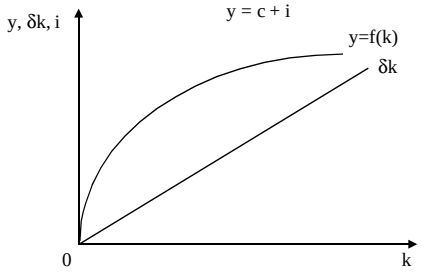

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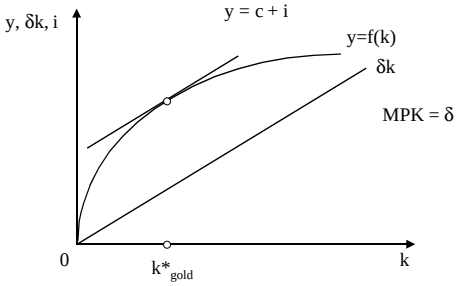

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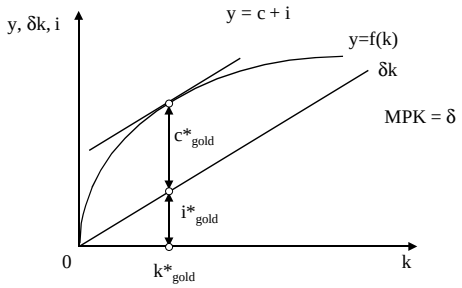

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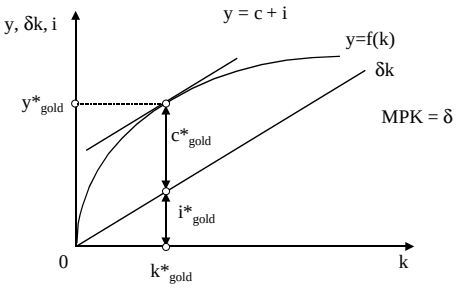

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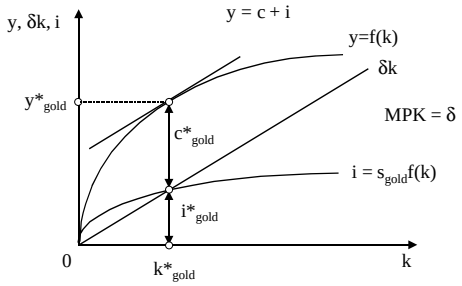

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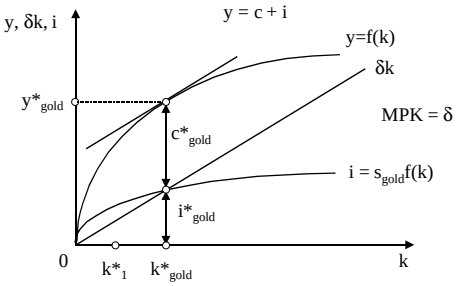

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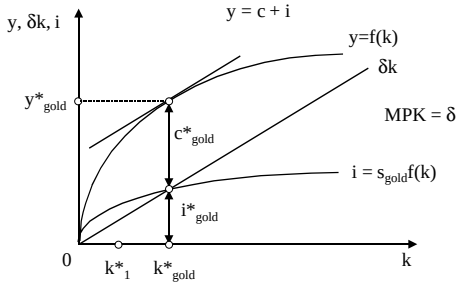

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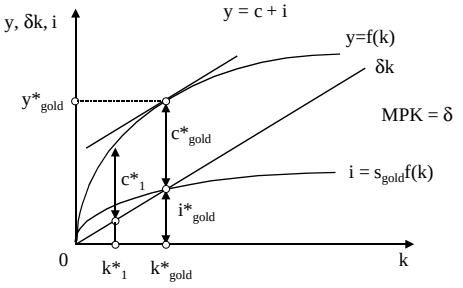

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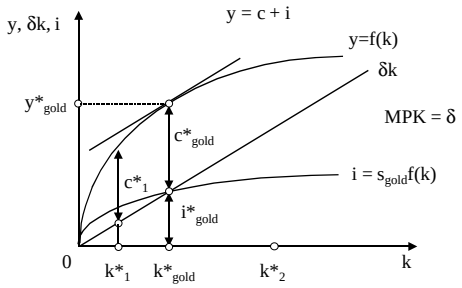

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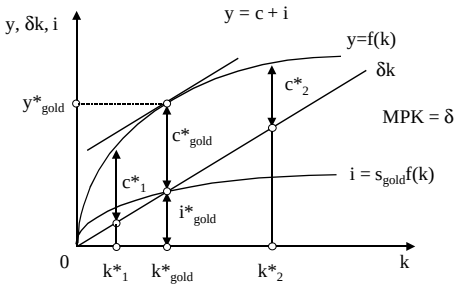
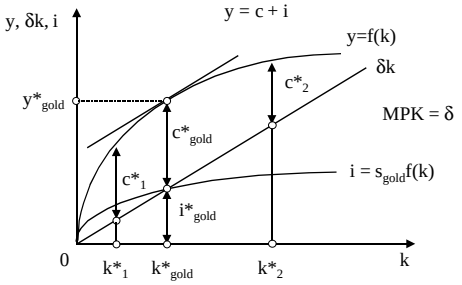


The Golden Rule: policy to get there

- k^*_{gold} is that steady state k which maximises steady state c
- Found where MPK = δ
- Consider when economy not at k^*_{gold}
- Economy is **dynamically inefficient**
- Two cases:
- $k^* < k^*_{gold}$
- $k^* > k^*_{gold}$
- What sort of policy problem does the government face?

Getting to the golden rule: either from k^*_1 or from k^*_2



Solow Growth Model: Full Version population growth + technological change

- Y = F(K, AL)
- Y output
- K capital
- L labour
- A the efficiency of labour
- so AL = the effective labour force
- Assume $\Delta L/L = n$
- Assume $\Delta A/A = g$
- both n and g are exogenous (rather than endogenous)

Solow Growth Model: Full Version population growth + technological change

- Define y = Y/(AL) output per unit of effective labour
- and k = K/(AL) …ditto for i and for c
- Given constant returns to scale
- y = f(k)
- i = sf(k)
- Fundamental equation of capital accumulation:

$$\Delta k = i - (\delta + n + g)k$$

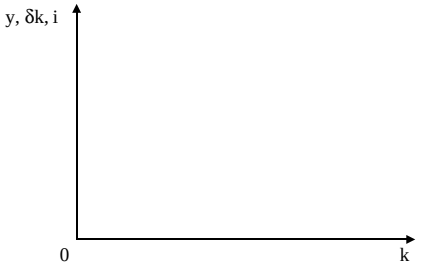
or

$$\Delta k = sf(k) - (\delta + n + g)k$$

a digression on growth rates

- If P and Q grow at rates p and q:
- (PQ) grows (approx.) at a rate (r + q) [product]
- (P/Q) grows (approx.) at a rate (p - q) [quotient]

Solow Growth Model: Steady-state in the full version



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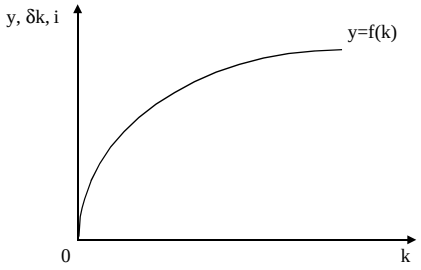

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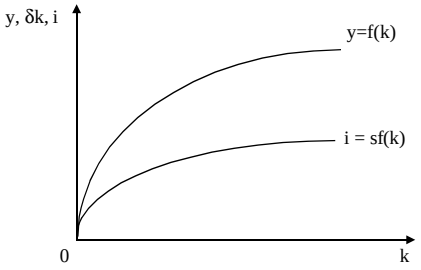

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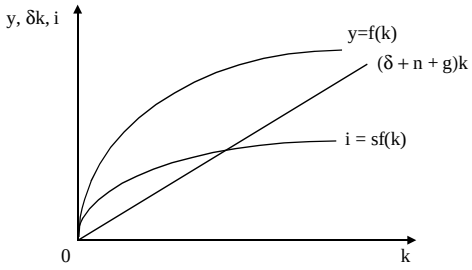

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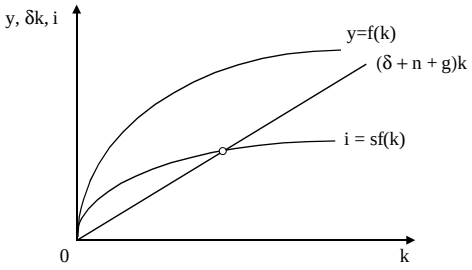

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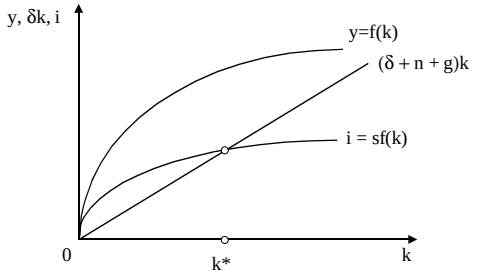

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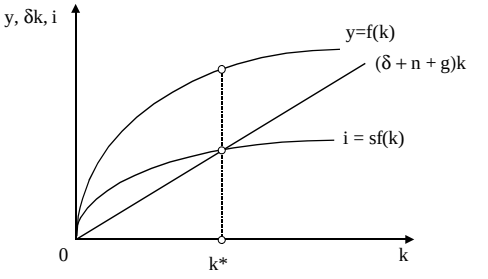

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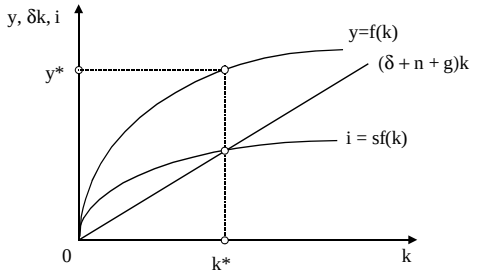

Solow growth model Steady-state in the full version

- y = Y/(AL) growing at rate 0 [from graph]
- Given that A is growing at rate g..
- and L is growing at rate n [by assumption]
- then, AL growing at rate n+g
- and Y growing at rate n+g
- [i.e. growth in total GDP]
- so Y/L growing at rate (n+g)-n = g
- i.e. technological progress only source of long-run growth in income per capita

The Solow Decomposition

- Given constant returns to scale
- and α = share of capital in income
- Then, growth rate of GDP =
- contribution of capital + contribution of labour + technological progress
- $\Delta Y/Y = \alpha\Delta K/K + (1-\alpha)\Delta L/L + \Delta A/A$
- Y, K, L, α observable
- $\Delta A/A$ Solow residual, total factor productivity

The Solow Decomposition

United States 1960-90

GNP	Capital	Labour	TFP
3.1%	0.9%	1.2%	1.1%

Newer models of growth

- Failure of convergence hypothesis
- Solow model over-simplifies technological progress
- This key factor is exogenous to the model
- Human capital
- Endogenous growth
  - e.g. externalities
- Institutional explanations